Homework #11
Truck Loading and Unloading

from Quantitive Methods for Business – Anderson, Sweeney and Williams

Trucks using a single-channel loading dock arrive according to a Poisson distribution. The time required to load/unload follows an exponential probability distribution. The mean arrival rate is 20 trucks per day, and the mean service rate is 30 trucks per day.

- ***Parameters*** are
  - **Mean Arrival Rate**
  - **Mean Service Rate**
- Queue type is / /

a. What is the probability that no trucks are in the system? One truck?

$$P_0 = 1 - (\lambda / \mu)$$

$$P_n = (1 - (\lambda / \mu))(\lambda / \mu)^n$$

b. What is the average number of trucks waiting for service? In the queue?

$$L = \lambda / (\mu - \lambda)$$

$$L_q = \lambda^2 / (\mu(\mu - \lambda))$$

c. What is the average time a truck waits for the loading/unloading service to begin? What is the average total time a truck spends in the system?

$$W_q = \lambda / (\mu(\mu - \lambda))$$

$$W = 1 / (\mu - \lambda)$$

d. What is the probability that an arriving truck will have to wait for service?
d. is looking for the calculation of P_w the probability that all the servers are busy.

$$P_w = \lambda / \mu$$

Finally what is the utilization rate r for the server?

$$\rho = \lambda / \mu$$

Yippee!! Expansion and More Profits!!!!!

Due to the incredible business acumen of the company's Management Scientists the number of loading docks has increased to 3; the mean arrival rate is 30 trucks per day, and the mean service rate on each line is 18 trucks per day.

- ***Parameters*** are
  - **Mean Arrival Rate –**
  - **Mean Service Rate –**
  - **Number of Servers –**
- Queue type is / /

a. What is the probability that no trucks are in the system? One truck?

$$P_0 = \frac{1}{\left[\sum_{n=1}^{n=k-1} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n\right] + \frac{1}{k!}\left(\frac{\lambda}{\mu}\right)^k \frac{k\mu}{k\mu - \lambda}}$$

$$P_n = \begin{cases} \dfrac{\left(\lambda/\mu\right)^n}{n!} P_0 \text{ for n } \leq \text{ k} \\ \dfrac{\left(\lambda/\mu\right)^n}{k!\,k^{(n-k)}} P_0 \text{ for n } > \text{ k} \end{cases}$$

b. What is the average number of trucks waiting for service? In the system?

$$L = \frac{\left(\lambda/\mu\right)^k \lambda\mu}{(k-1)!\left(k\mu-\lambda\right)^2} P_0 + \frac{\lambda}{\mu}$$

$$L_q = \frac{\left(\lambda / \mu\right)^k \lambda\mu}{(k-1)!\left(k\mu - \lambda\right)^2} P_0$$

c. What is the average time a truck waits for the loading/unloading service to begin? What is the average total time a truck spends in the system?

$$W = \frac{\left(\lambda/\mu\right)^k \mu}{(k-1)!\left(k\mu-\lambda\right)^2} P_0 + \frac{1}{\mu}$$

$$W_q = \frac{\left(\lambda/\mu\right)^k \mu}{(k-1)!\left(k\mu-\lambda\right)^2} P_0$$

d. What is the probability that an arriving truck will have to wait for service?

d. is looking for the calculation of P_w the probability that all the servers are busy.

$$P_w = \frac{1}{k!}\left(\frac{\lambda}{\mu}\right)^k \left(\frac{k\mu}{k\mu - \lambda}\right) P_0$$

Finally what is the utilization rate r for the server?

$$\rho = \lambda / k\mu$$