UNIVERSITY OF MASSACHUSETTS

Isenberg School of Management

Department of Finance and Operations Management

FOMGT 353-Introduction to Management Science

Homework #2 – Graphical LP's.

Show your work completely and in an organized manner to receive maximum credit. Correct answers without supporting calculations or diagrams will not receive credit. Incorrect answers using the correct method and a good presentation will receive substantial credit.

My name is:

1. Consider the following linear program:

   Maximize x + 2y

   Subject to

   x <= 5 (1)

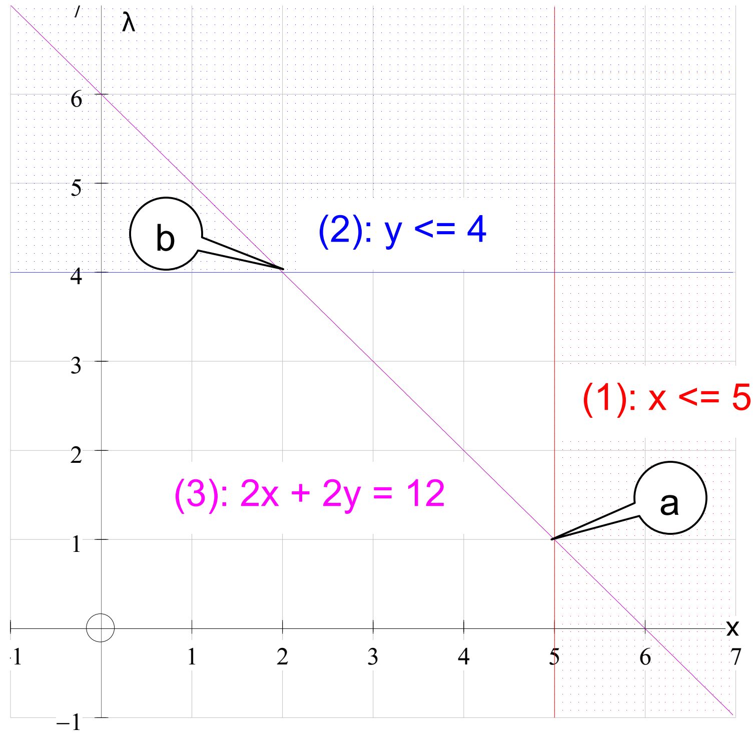
   y <= 4 (2)

   2x + 2y = 12 (3)

   x => 0, y => 0

a. Graph the constraints and bounds.

   Work out the size of the graph to make it easier to draw! Constraints (1) and (2) are easy. Setting x = 0 in (3) gives 2y = 12 or y = 6. Similarly setting y = 0 in (3) gives 2x = 12 or x = 6. Max x is 6 and Max y is 6…



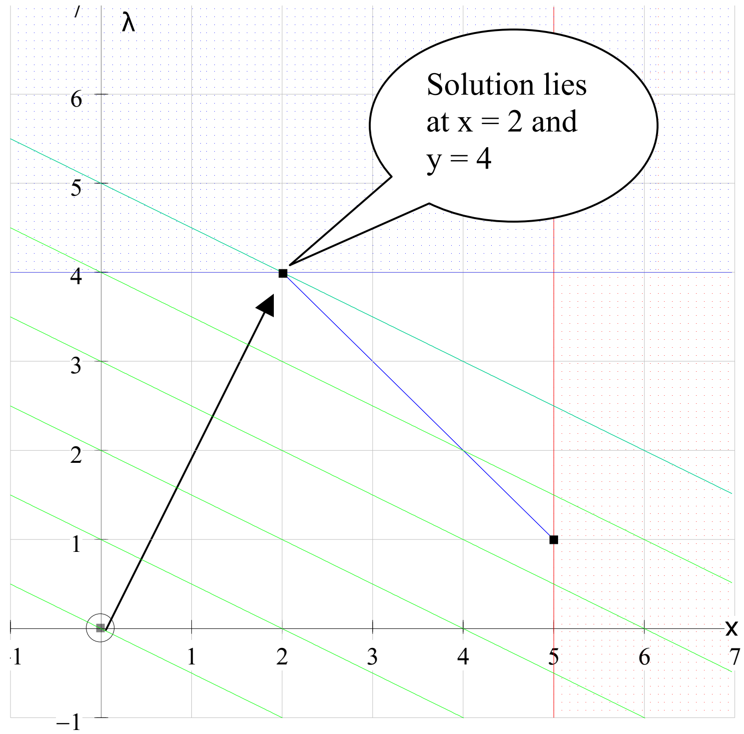
b. Identify the feasible region.

The line segment on constraint (3) between point a (x=5, y=2) and point b (x=2, y=4).

c. What are the extreme points of the feasible region?

The extreme points of the feasible region are points a and b.

d. Find the optimal solution using the graphical procedure.



We can read the Optimal Values of the decision variables off the Graph in this case. However we can solve the simultaneous equations (2) y = 4 and (3) 2x + 2y = 12. Clearly y = 4 which is substituted into (3) giving 2x + 2(4) = 12

2x = 12 – 8

x = 4/2 = 2

We obtain the Objective Function Value by substituting these values into the Objective Function x + 2y
giving 2 + 2(4) = 2 + 8 = 10

2. Embassy Motorcycles (EM) manufacturers two lightweight motorcycles designed for easy handling and safety. The EZ-Rider model has a new engine and a low profile that make it easy to balance. The Lady-Sport model is slightly larger, uses a more traditional engine and is specifically designed to appeal to women riders.

   Embassy produces the engines for both models at its Des Moines, Iowa, plant. Each EZ-Rider engine requires 6 hours of manufacturing time and each Lady-Sport engine requires 3 hours of manufacturing time. The Des Moines plant has 2100 hours of engine manufacturing time available for the next production period.

   Embassy's motorcycle frame supplier can supply as many EZ-Rider frames as needed. However, the Lady-Sport frame is more complex and the supplier can only provide up to 280 Lady-Sport frames for the next production period.

Final assembly and testing requires 2 hours for each EZ-Rider model and 2.5 hours for each Lady-Sport model. A maximum of 1000 hours of assembly and testing time are available for the next production period.

The company's accounting department projects a profit contribution of $2400 for each EZ-Rider produced and $1800 for each Lady-Sport produced.

a. Formulate a linear programming model that can be used to determine the number of units of each model that should be produced in order to maximize the total contribution to profit.

   Let x1 be number of EZ-Riders produced.
   Let x2 be number of Lady-Sports produced

   The Objective is to maximize the profit contribution:

   Max 2,400 x1 + 1,800 x2

   The Constraints are:
   Engine Manufacturing Time <= 2100 hours
   Lady-Sport Frames <= 280 frames
   Assembly & Test <= 1000 hours

6 x1 + 3 x2	<= 2100	(Engine Manufacturing)
x2	<= 280	(Lady-Sport Frames)
2 x1 + 2.5 x2	<= 1000	(Assembly & Test)

   We also need bounds x1 and x2 => 0.

   So the full model is:

   Max 2,400 x1 + 1,800 x2
   Subject to

6 x1 + 3 x2	<= 2100	(Engine Manufacturing)
x2	<= 280	(Lady-Sport Frames)
2 x1 + 2.5 x2	<= 1000	(Assembly & Test)

   x1 => 0, x2 =>0

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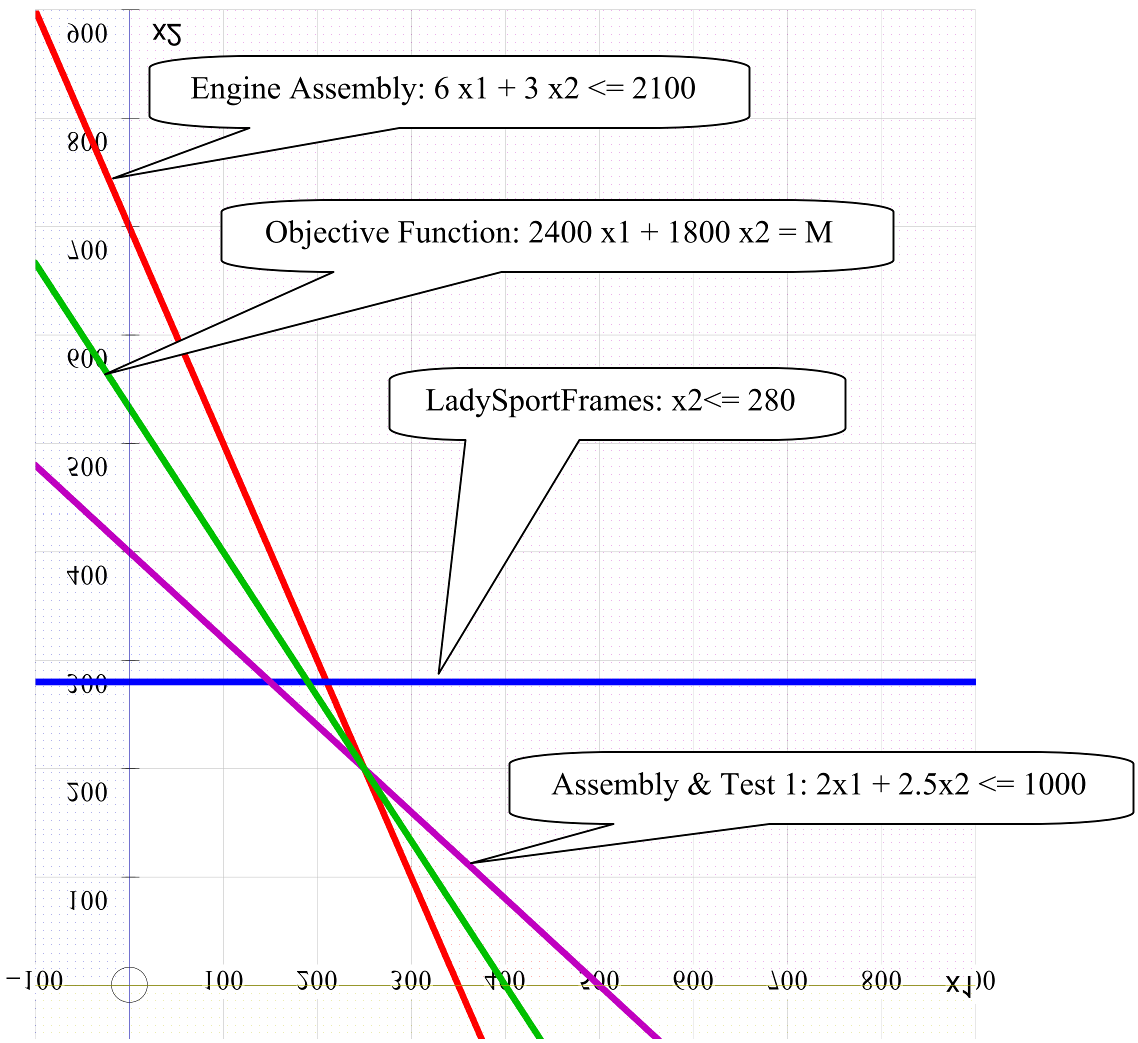
b. Solve the problem graphically. What is the optimal solution?

x1 is 350 if x2 = 0 and x2 = 700 if x1 = 0 in Engine Manufacturing
x1 is max 280 in Lady-Sports Frames
x1 is 500 if x2 = 0 and x2 = 400 if x1 = 0 in Assembly & Test

So we need to allow up to 800 for x2 and up to 500 for x1 on our graph.



So the solution occurs where:

a. $2 x1 + 2.5 x2 = 1000$ and
b. $6 x1 + 3 x2 = 2100$

So we solve these simultaneous equations:

Set $x2 = (2100/3) - (6/3) x1$ from equation b.

Substitute into equation a.
$2 x1 + (5/2) *((2100/3) -(2 x1)) =1000$
$2 x1 + (10500/6) -5 x1 = 1000$
$-3 x1 =1000 - 1750$
$3 x1 = 750$
$x1 = 250$

Substitute into equation b.
$(6 * 250) + 3 x2 = 2100$
$3 x2 = 2100 - 1500$
$x2 = 200$

The Objective Function value is $2400 x1 + 1800 x2$

$= (2400 * 250) + (1800 * 200)$
$= 600{,}000 + 360{,}000$
$= 960{,}000$

So the solution is:

$x1 = 250$, $x2 = 200$ and the Objective Function value $Z= 960{,}000$

c. Which constraints are binding?

Assembly & Test and Engine Manufacturing are the binding constraints.