UNIVERSITY OF MASSACHUSETTS

Isenberg School of Management

Department of Finance and Operations Management

FOMGT 353-Introduction to Management Science

SOLUTION - Homework #3 – The Simplex Method.

The Fred Basset Candy Company makes Fudge and Chocolate Bars. They have 100 lbs of Molasses, 36 lbs of Cocoa and 60 liters of Milk on hand and they need to decide which products to make, today, and in what quantities to make the highest contribution to profits.

A Fudge Bar uses a pound of Molasses, .3 of a pound of Cocoa and .2 liters of Milk. Only .35 lbs of Molasses, .2 lbs of Cocoa and half a liter of milk are needed for a Chocolate Bar.

A pound of Molasses costs $1., a pound of Cocoa costs 50 cents and a liter of milk 80 Cents. A Fudge Bar is sold for $1.61 and a Chocolate Bar for $1.

1. Formulate the Linear Program

The objective is to maximize the contribution from sales of Fudge and Chocolate bars. But we are not given what they are so calculate these values first.

Contribution = Sales Price – Variable Cost

Variable Cost = Materials Used * Costs

Contribution from Fudge bar = $1.61 –(1 * $1) – (0.3 * $0.5) – (0.2 * $0.8) = $0.3

Contribution from Choc bar = $1 –(0.35 * $1) – (0.2 * $0.5) – (0.5 * $0.8) = $0.15

Let x1 be the number of Fudge Bars produced, and
Let x2 be the number of Chocolate Bars produced.

The constraints are the amount of raw materials on hand.

- Amount of Molasses used <= 100 lbs
- Amount of Cocoa used <= 36 lbs
- Amount of Milk used <= 60 litres

A Fudge bar needs 1lb of Molasses and a Chocolate bar needs 0.35 lbs, so if we make x1 and x2 of each respectively, the amount used will be:
$(x1 * 1) + (x2 * 0.35) = x1 + 0.35\ x2$

and the constraint is $x1 + 0.35\ x2 <= 100$ (Molasses)

Similarly $0.3\ x1 + 0.2\ x2 <= 36$ (Cocoa)
And $0.2\ x1 + 0.5\ x2 <= 60$ (Milk)

We cannot turn candy back into their ingredients so we need bounds

$x1 => 0$ and $x2 => 0$

So the model is:

$$
\begin{array}{lll}
\text{Max} & 0.30\ x1 + 0.15\ x2 & \\
\text{Subject to} & & \\
 & x1 + 0.35\ x2 <= 100 & \text{(Molasses)} \\
 & 0.3\ x1 + 0.20\ x2 <= 36 & \text{(Cocoa)} \\
 & 0.2\ x1 + 0.50\ x2 <= 60 & \text{(Milk)} \\
 & x1, x2 => 0 &
\end{array}
$$

2. Convert the LP to **Standard Form.**

We have <= inequalities so we can add slack variable to each constraint:

$$
\begin{array}{lll}
\text{Max} & 0.30\ x1 + 0.15\ x2 & \\
\text{Subject to} & & \\
 & x1 + 0.35\ x2 + s1 = 100 & \text{(Molasses)} \\
 & 0.3\ x1 + 0.20\ x2 + s2 = 36 & \text{(Cocoa)} \\
 & 0.2\ x1 + 0.50\ x2 + s3 = 60 & \text{(Milk)} \\
 & x1, x2, s1, s2, s3 => 0 & \text{(Bounds)}
\end{array}
$$

3. Convert the LP to **Canonical Form.**

We have equalities, positive RHS values and s1, s2 and s3 each appear in only one constraint and have "+1" coefficients so the LP is already in Canonical Form.

4. What is the **Initial Basic Feasible Solution**?

$x1 = 0, x2 = 0, s1 = 100, s2 = 36, s3 = 60$ and the objective function value Z is zero.

5. Construct the Initial Simplex Tableau.

		x1	x2	s1	s2	s3	
Basis	Cj	0.3	0.15	0	0	0	Bi
s1	0	1	0.35	1	0	0	100
s2	0	0.3	0.2	0	1	0	36
s3	0	0.2	0.5	0	0	1	60
	Zj	0	0	0	0	0	0
	Cj - Zj	0.3	0.15	0	0	0	

6. Which Variable is the Entering Variable and which variable is the Leaving Variable at this point in the Simplex Method.

The Maximum (Cj - Zj) is 30 so x1 is the Entering Variable

x1 can increase by 100/1 if s1 decreases to zero	or	100
x1 can increase by 36/.3 if s2 decreases to zero	or	120
x1 can increase by 60/.2 if s3 decreases to zero	or	300

The maximum x1 we can achieve without setting a Basic Variable to a negative number is 100.
So s1 will be the Leaving Variable and Row 1 and Column 1 will define the Pivot.

7. Solve the Problem using the Simplex Method

Basis	Cj	x1	x2	s1	s2	s3	Bi
		0.3	0.15	0	0	0	
x1	0	1	0.35	1	0	0	100
s2	0	0.3	0.2	0	1	0	36
s3	0	0.2	0.5	0	0	1	60
	Zj	0	0	0	0	0	0
	Cj - Zj	0.3	0.15	0	0	0	

(A) Row One divided by pivot element	1	0.35	1	0	0	100
(B) Row (A) * Pivot Column Element in s2 Row	0.3	0.105	0.3	0	0	30
(C) s2 Row -Row (B)	0	0.095	-0.3	1	0	6
(D) Row (A) * Pivot Column Element in s3 Row	0.2	0.07	0.2	0	0	20
(E) S3 Row - Row (D)	0	0.43	-0.2	0	1	40

The Pivot Element Value is 1 (in Yellow).
The Pivot Column Element in s2 Row Value is 0.3 (in Orange)
The Pivot Column Element in s3 Row Value is 0.2 (in Purple)
Transfer the new rows to a new tableau, update the basis, recalculate Zj and (Cj-Zj) values.

Basis	Cj	x1	x2	s1	s2	s3	Bi
		0.3	0.15	0	0	0	
x1	0.3	1	0.35	1	0	0	100
s2	0	0	0.095	-0.3	1	0	6
s3	0	0	0.43	-0.2	0	1	40
	Zj	0.3	0.105	0.3	0	0	30
	Cj - Zj	0	0.045	-0.3	0	0	

The Maximum (Cj - Zj) is 0.045 so x2 is the Entering Variable

x2 can increase by 100/.35 if x1 decreases to zero	or	285.714286
x2 can increase by 6/.095 if s2 decreases to zero	or	63.1578947
x2 can increase by 40/.43 if s3 decreases to zero	or	93.0232558

The maximum x2 we can achieve without setting a Basic Variable to a negative number is 63.157895.
So s2 will be the Leaving Variable and Row 2 and Column 2 will define the Pivot.

		x1	x2	s1	s2	s3	
Basis	Cj	0.3	0.15	0	0	0	Bi
x1	0.3	1	0.35	1	0	0	100
s2	0	0	0.095	-0.3	1	0	6
s3	0	0	0.43	-0.2	0	1	40
	Zj	0.3	0.105	0.3	0	0	30
	Cj - Zj	0	0.045	-0.3	0	0	

(A) Row Two divided by pivot element	0	1	-3.16	10.53	0	63.158
(B) Row (A) * Pivot Column Element in x1 Row	0	0.35	-1.11	3.684	0	22.105
(C) x1 Row -Row (B)	1	0	2.105	-3.68	0	77.895
(D) Row (A) * Pivot Column Element in s3 Row	0	0.43	-1.36	4.526	0	27.158
(E) S3 Row - Row (D)	0	0.00	1.158	-4.53	1	12.842

		x1	x2	s1	s2	s3	
Basis	Cj	0.3	0.15	0	0	0	Bi
x1	0.3	1	0	2.105	-3.68	0	77.895
x2	0.15	0	1	-3.16	10.53	0	63.158
s3	0	0	0	1.158	-4.53	1	12.842
	Zj	0.3	0.15	0.158	0.474	0	32.842
	Cj - Zj	0	0	-0.16	-0.47	0	

The Maximum (Cj - Zj) is -.16 so there is no Entering Variable

We have found the optimal solution, which is:

X1 = 77.895
X2 = 63.158
S3 = 12.842
S1 = 0
S2 = 0
And
Z = 32.842