UNIVERSITY OF MASSACHUSETTS

Isenberg School of Management

Department of Finance and Operations Management

FOMGT 353-Introduction to Management Science

Homework #4 – The Simplex Method.

Show your work completely and in an organized manner to receive maximum credit. Correct answers without supporting calculations or diagrams will not receive credit. Incorrect answers using the correct method and a good presentation will receive substantial credit.

My name is:

Solve the following linear program in the steps outlined in parts 1. to 6.

Minimize 152 x1 + 140 x2
(1): 1.2 x1 + 1.4 x2 >= 10
(2): 1.4 x1 + 0.8 x2 >= 10
(3): 1.5 x1 + 0.2 x2 >= 7.1
(4): -x1 - x2 => -15
x1, x2 =>0

1. Convert the LP to Standard Form.

Minimize 152 x1 + 140 x2
Subject to
(1): 1.2 x1 + 1.4 x2 - s1 = 10
(2): 1.4 x1 + 0.8 x2 - s2 = 10
(3): 1.5 x1 + 0.2 x2 - s3 = 7.1
(4): x1 + x2 + s4 = 15
x1, x2, s1, s2, s3, s4 =>0

2. Convert the LP to Canonical Form.

Minimize 152 x1 + 140 x2 + M*a1 + M*a2 + M*a3
Subject to
(1): 1.2 x1 + 1.4 x2 - s1 + a1 = 10
(2): 1.4 x1 + 0.8 x2 - s2 + a2 = 10
(3): 1.5 x1 + 0.2 x2 -s3 + a3 = 7.1
(4): x1 + x2 + s4 = 15
x1, x2, s1, s2, s3, s4, a1, a2, a3 =>0

3. What is the Initial Basic Feasible Solution?

x1 = 0, x2 = 0, s1 = 0, s2 = 0, s3 = 0, s4 = 15, a1 = 10, a2 = 10, a3 = 7.1 and z = 27.1M

4. Construct the Initial Simplex Tableau. Let M = 1,000 to keep things simple!!

5. Which Variable is the Entering Variable and which variable is the Leaving Variable at this point in the Simplex Method.

		x1	x2	s1	s2	s3	s4	a1	a2	a3		
Basis	Cj	152	140	0	0	0	0	1000	1000	1000	Bi	
a1	1000	1.2	1.4	-1	0	0	0	1	0	0	10	8.33
a2	1000	1.4	0.8	0	-1	0	0	0	1	0	10	7.14
a3	1000	1.5	0.2	0	0	-1	0	0	0	1	7.1	4.73
s4	0	1	1	0	0	0	1	0	0	0	15	15.00
	Zj	4100	2400	-1000	-1000	-1000	0	1000	1000	1000	27100	
	Cj-Zj	-3948	-2260	1000	1000	1000	0	0	0	0		

Most negative Cj-Zj, so x1 is the Entering Variable.

The minimum positive Ratio is 4.73333, so a3 is the Leaving Variable.

6. Solve the problem in 3 pivots, and give the Optimal Solution in full.

New Row 3	x1	152	1	0.133	0.000	0.000	-0.667	0.000	0.000	0.000	0.667	4.733
New Row 1	a1	1000	0	1.240	-1.000	0.000	0.800	0.000	1.000	0.000	-0.800	4.320
New Row 2	a2	1000	0	0.613	0.000	-1.000	0.933	0.000	0.000	1.000	-0.933	3.373
New Row 4	s4	0	0	0.867	0.000	0.000	0.667	1.000	0.000	0.000	-0.667	10.267

		x1	x2	s1	s2	s3	s4	a1	a2	a3		
Basis	Cj	152	140	0	0	0	0	1000	1000	1000	Bi	
a1	1000	0	1.240	-1.000	0.000	0.800	0.000	1.000	0.000	-0.800	4.320	3.4839
a2	1000	0	0.613	0.000	-1.000	0.933	0.000	0.000	1.000	-0.933	3.373	5.5
x1	152	1	0.133	0.000	0.000	-0.667	0.000	0.000	0.000	0.667	4.733	35.5
s4	0	0	0.867	0.000	0.000	0.667	1.000	0.000	0.000	-0.667	10.267	11.846
	Zj	152	1874	-1000	-1000	1632	0	1000	1000	-1632	8413	
	Cj-Zj	0	-1734	1000	1000	-1632	0	0	0	2632		

Most negative Cj-Zj, so x2 is the Entering Variable.

The minimum positive Ratio is 3.483871, so a1 is the Leaving Variable.

New Row 1	x2	140	0	1	-0.806	0	0.6452	0	0.8065	0	-0.645	3.4839
New Row 2	a2	M	0	0	0.4946	-1	0.5376	0	-0.495	1	-0.538	1.2366
New Row 3	x1	152	1	0	0.1075	0	-0.753	0	-0.108	0	0.7527	4.2688
New Row 4	s4	0	0	0	0.6989	0	0.1075	1	-0.699	0	-0.108	7.2473

		x1	x2	s1	s2	s3	s4	a1	a2	a3		
Basis	Cj	152	140	0	0	0	0	1000	1000	1000	Bi	
x2	140	0	1.000	-0.806	0.000	0.645	0.000	0.806	0.000	-0.645	3.484	5.4
a2	1000	0	0.000	0.495	-1.000	0.538	0.000	-0.495	1.000	-0.538	1.237	2.3
x1	152	1	0.000	0.108	0.000	-0.753	0.000	-0.108	0.000	0.753	4.269	-5.671
s4	0	0	0.000	0.699	0.000	0.108	1.000	-0.699	0.000	-0.108	7.247	67.4
	Zj	152	140	398	-1000	514	0	-398	1000	-514	2373	
	Cj-Zj	0	0	-398	1000	-514	0	1398	0	1514		

Most negative Cj-Zj, so s3 is the Entering Variable.

The minimum positive Ratio is 2.3, so a2 is the Leaving Variable.

New Row 2	s3	0	0	0	0.92	-1.86	1	0	-0.92	1.86	-1	2.3
New Row 1	x2	140	0	1	-1.4	1.2	0	0	1.4	-1.2	0	2
New Row 3	x1	152	1	0	0.8	-1.4	0	0	-0.8	1.4	0	6
New Row 4	s4	0	0	0	0.6	0.2	0	1	-0.6	-0.2	0	7

		x1	x2	s1	s2	s3	s4	a1	a2	a3	
Basis	Cj	152	140	0	0	0	0	1000	1000	1000	Bi
x2	140	0	1.000	-1.400	1.200	0.000	0.000	1.400	-1.200	0.000	2.000
s3	0	0	0.000	0.920	-1.860	1.000	0.000	-0.920	1.860	-1.000	2.300
x1	152	1	0.000	0.800	-1.400	0.000	0.000	-0.800	1.400	0.000	6.000
s4	0	0	0.000	0.600	0.200	0.000	1.000	-0.600	-0.200	0.000	7.000
	Zj	152.0	140.0	-74.4	-44.8	0.0	0.0	74.4	44.8	0.0	1192.0
	Cj-Zj	0	0	74	45	0	0	926	955	1000	

No negative Cj-Zj values so stop solution is optimal at x1 = 6, x2 = 2, s1 = 0, s2 = 0, s3 - 2.3, s4 = 7 and with an Objective Function Value z = 1,192