LP Applications III

The Diet Problem, And Inventory Holding Over Time Periods

The Diet Problem

- The following foodstuffs contain vitamins A, B, C, and D. They also have a calorific content, a "Bad Cholesterol" content and a price per pound. The objective is to ensure a good diet defined as certain daily minimum levels of the vitamins and calories as well as certain daily maximum levels of calories and "Bad Cholesterol".

Foodstuff	A	B	C	D	"Bad Cholesterol"	Calories	Price Per lb.
Eggs	12	2	0	2	150	250	$0.75
Beef	10	12	0	40	200	500	$4.60
Bread	5	15	0	2	25	125	$3.00
Apples	12	7	200	0	0	90	$2.30
Spinach	0	5	1000	20	0	150	$2.50
Min Daily	100	120	2000	85	0	1800	
Max Daily					500	3000	

The Diet Problem Cont…

- What is the recommended minimum cost diet based on these criteria?
- Changing the price of Apples only, within what limits does the solution remain the same?
- We decide we can get away with only 1,600 calories a day. How does the solution change?
- We decide to increase the maximum level of "Bad Cholesterol" from 500 to 1,500 does the Sensitivity analysis allow us to predict the change in the Objective Function Value?
- If we decrease the maximum level of "Bad Cholesterol" from 400 to 500, what is the effect on the Objective Function Value?
- Could you manage this diet? What would you do to the model to make the result more realistic?

The Decision Variables and The Objective Function

- We are trying to decide how much of each foodstuff to include in our diet, so the Decision Variables are:
  - "Eggs", "Beef", "Bread", "Apples" and "Spinach".
- The Objective is to minimize the cost:
  - Min 0.75 Eggs + 4.6 Beef + 3.0 Bread + 2.30 Apples + 2.50 Spinach.

Constraints

- Our diet requires minimum levels of Vitamins and Calories and maximum levels of "Bad Cholesterol" and Calories, so these are the constraints:
  - Min A. 12 Eggs + 10 Beef + 5 Bread + 12 Apples + 0 Spinach => 100
  - Min B. 2 Eggs + 12 Beef + 15 Bread + 7 Apples + 5 Spinach => 120
  - Min C. 0 Eggs + 0 Beef + 0 Bread + 200 Apples + 1,000 Spinach => 2,000
  - Min D. 2 Eggs + 40 Beef + 2 Bread + 0 Apples + 20 Spinach => 85
  - Max "Bad". 150 Eggs + 200 Beef + 25 Bread + 0 Apples + 0 Spinach <= 500
  - Min Calories. 250 Eggs + 500 Beef + 125 Bread + 90 Apples + 150 Spinach => 1,800
  - Max Calories. 250 Eggs + 500 Beef + 125 Bread + 90 Apples + 150 Spinach => 1,800

The Model In Excel

	A	B	C	D	E	F	G	H	I	J	K
1	**Objective**										
2											
3	Min Cost	$0.00									
4											
5	**Decision Variables**			Foodstuff	A	B	C	D	"Bad Cholesterol"	Calories	Price Per lb.
6	Eggs	0		Eggs	12	2	0	2	150	250	$0.75
7	Beef	0		Beef	10	12	0	40	200	500	$4.60
8	Bread	0		Bread	5	15	0	2	25	125	$3.00
9	Apples	0		Apples	12	7	200	0	0	90	$0.00
10	Spinach	0		Spinach	0	5	1000	20	0	150	$2.50
11				Min Daily	100	120	2000	85	0	1800	
12				Max Daily					500	3000	
13											
14	**Constraints**										
15											
16	Min A	0	=>	100							
17	Min B	0	=>	120							
18	Min C	0	=>	2000							
19	Min D	0	=>	85							
20	Max Bad	0	<=	500							
21	Min Calories	0	=>	1800							
22	Max Calories	0	<=	3000							
23											

=($B$6*K6)+($B$7*K7)+($B$8*K8)+($B$9*K9)+($B$10*K10)

=($B$6*E6)+($B$7*E7)+($B$8*E8)+($B$9*E9)+($B$10*E10)

The Answer Report

Microsoft Excel 9.0 Answer Report
Worksheet: [DietProblem.xls]Diet Problem
Report Created: 1/26/2002 4:42:13 PM

Target Cell (Min)

Cell	Name	Original Value	Final Value
$B$3	Min Cost	$0.00	$32.96

Adjustable Cells

Cell	Name	Original Value	Final Value
$B$6	Eggs	0	0.8461835
$B$7	Beef	0	1.324209715
$B$8	Bread	0	4.32922128
$B$9	Apples	0	4.579799537
$B$10	Spinach	0	1.084040093

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$B$16	Min A	100	$B$16>=$D$16	Binding	0
$B$17	Min B	120	$B$17>=$D$17	Binding	0
$B$18	Min C	2000	$B$18>=$D$18	Binding	0
$B$19	Min D	85	$B$19>=$D$19	Binding	0
$B$20	Max Bad	500	$B$20<=$D$20	Binding	0
$B$21	Min Calories	1989.591365	$B$21>=$D$21	Not Binding	189.5913647
$B$22	Max Calories	1989.591365	$B$22<=$D$22	Not Binding	1010.408635

The Min Calories Constraint is not binding so the model will be unchanged if we reduce it to 1,600!

- So we are going to eat 4.57 lbs of Apples and 4.32 lbs of Bread!!
- The total cost will be $32.96!! (shows what Andrew knows about food prices!!)

The Sensitivity Report

Microsoft Excel 9.0 Sensitivity Report
Worksheet: [DietProblem.xls]Diet Problem
Report Created: 1/26/2002 4:42:13 PM

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$6	Eggs	0.8461835	0	0.75	0.649741602	0.390414508
$B$7	Beef	1.324209715	0	4.6	0.502333333	2.228481013
$B$8	Bread	4.32922128	0	3	1.556965944	1.794047619
$B$9	Apples	4.579799537	0	2.3	0.88128655	0.63138732
$B$10	Spinach	1.084040093	0	2.5	3.156936598	0.272513562

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$B$16	Min A	100	0.087406836	100	52.85714286	44.66165414
$B$17	Min B	120	0.170431766	120	156.7857143	43.3686067
$B$18	Min C	2000	0.000290478	2000	2290	1016.998192
$B$19	Min D	85	0.067868157	85	23.60215054	43.48101266
$B$20	Max Bad	500	-0.005169879	500	559.3220339	106.3468992
$B$21	Min Calories	1989.591365	0	1800	189.5913647	1E+30
$B$22	Max Calories	1989.591365	0	3000	1E+30	1010.408635

We cannot say what would happen if the RHS of the Max Bad constraint were increased by 1,000 since the Allowable Increase is only 559!

The model is OK for predicting what happens if the RHS of the constraint is reduced by 100! The Shadow Price tells us that for every 1 decrease the costs will increase by 0.005169879

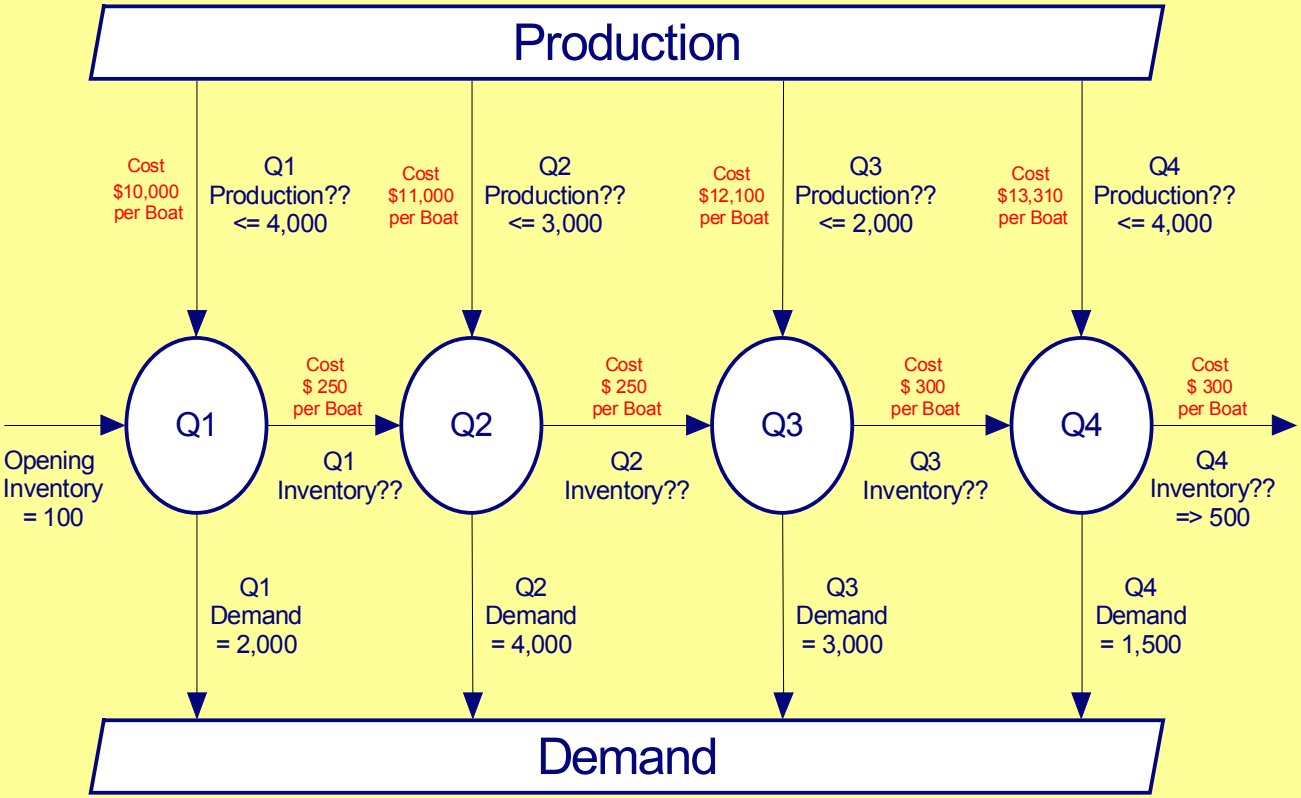
Making the Model More Realistic

- This much Bread and Apples would not be eatable!
  - So, place reasonable upper bounds on the amount of each foodstuff to be eaten, e.g.
    - Bread <= 1 and Apples <= 1
  - Add more foodstuffs to the mix and re-run the problem.

Inventory Holding Over Time Periods

- The production manager for the Classic Boat Corporation must determine how many units of the Classic 21 model to produce over the next four quarters.
- The company has a beginning inventory of 100 Classic 21 boats, and demand for the four quarters is 2000 units in quarter 1, 4000 units in quarter 2, 3000 units in quarter 3, and 1500 units in quarter 4.
- The firm has limited production capacity in each quarter. That is, up to 4000 units can be produced in quarter 1, 3000 units in quarter 2, 2000 units in quarter 3, and 4000 units in quarter 4.
- Each boat held in inventory in quarters 1 and 2 incurs an inventory holding cost of $250 per unit; the holding cost for quarters 3 and 4 is $300 per unit.
- The production costs for the first quarter are $10,000 per unit; these costs are expected to increase by 10% each quarter because of increases in labor and material costs.
- Management has specified that the ending inventory for quarter 4 must be at least 500 boats.

If In Doubt Draw a Picture!

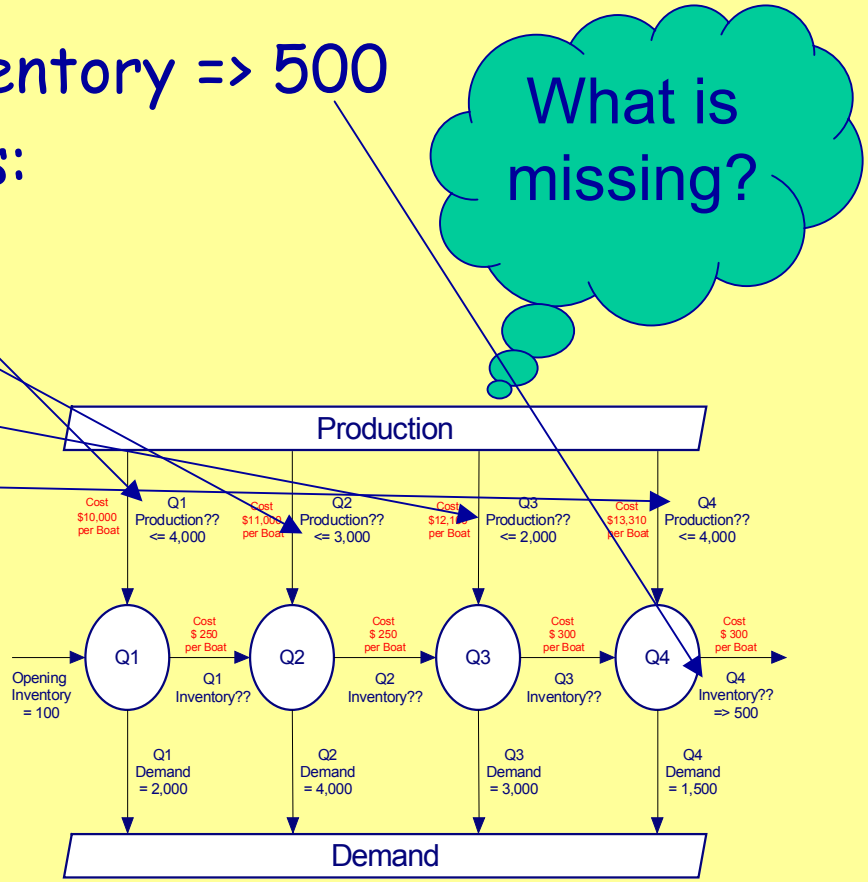


Decision Variables and Objective Function

- Hopefully the Decision Variables fall out of the picture as:
  - Q1Production, Q2Production, Q3Production, Q4Production, Q1Inventory, Q2Inventory, Q3Inventory, Q4Inventory.
- The Objective is to minimize the cost of production, by holding inventory, which itself has a cost! So…
  - Min 10,000 Q1Production + 11,000 Q2Production + 12,100 Q3Production + 13,310 Q4Production + 250 Q1Inventory + 250 Q2Inventory + 300 Q3Inventory + 300 Q4Inventory.

Constraints

- Closing Inventory: Q4Inventory => 500
- Qterly Production Ceilings:
  - Q1Production <= 4,000,
  - Q2 Production <= 3,000,
  - Q3Production <= 2,000 and
  - Q4Production <= 4,000.



Flow Conservation Constraints

- We need to leave nothing and take nothing from each Quarter! The sum of the incoming flows less the outgoing flows must equal zero!
  - Q0Inventory + Q1Production – Q1Demand – Q1Inventory = 0
  - Q1Inventory + Q2Production – Q2Demand – Q2Inventory = 0
  - Q2Inventory + Q3Production – Q3Demand – Q3Inventory = 0
  - Q3Inventory + Q4Production – Q4Demand – Q4Inventory = 0

The Model in Excel

	A	B	C	D	E	F	G	H	I
1	**Objective**								
2					=SUM(C7:C14)				
3	CostMin	47,510							
4									
5	**Decision Variables**				**Parameters**				
6			CostPer		Costs				
7	Q1Production	1	10000		10000		DemandQ1	=	2000
8	Q2Production	1	11000		11000		DemandQ2	=	4000
9	Q3Production	1	12100		12100		DemandQ3	=	3000
10	Q4Production	1	13310		13310		DemandQ4	=	1500
11	Q1Inventory	1	250		250		InventoryQ0	=	100
12	Q2Inventory	1	250		250				
13	Q3Inventory	1	300		300				
14	Q4Inventory	1	300		300				
15									
16	**Constraints**								
17									
18	InventoryQ4	1	=>		500				
19	ProductionQ1	1	<=		4000				
20	ProductionQ2	1	<=		3000				
21	ProductionQ3	1	<=		2000				
22	ProductionQ4	1	<=		4000				
23	ConservationQ1	-1900	=		0				
24	ConservationQ2	-3999	=		0				
25	ConservationQ3	-2999	=		0				
26	ConservationQ4	-1499	=		0				

Format Painter

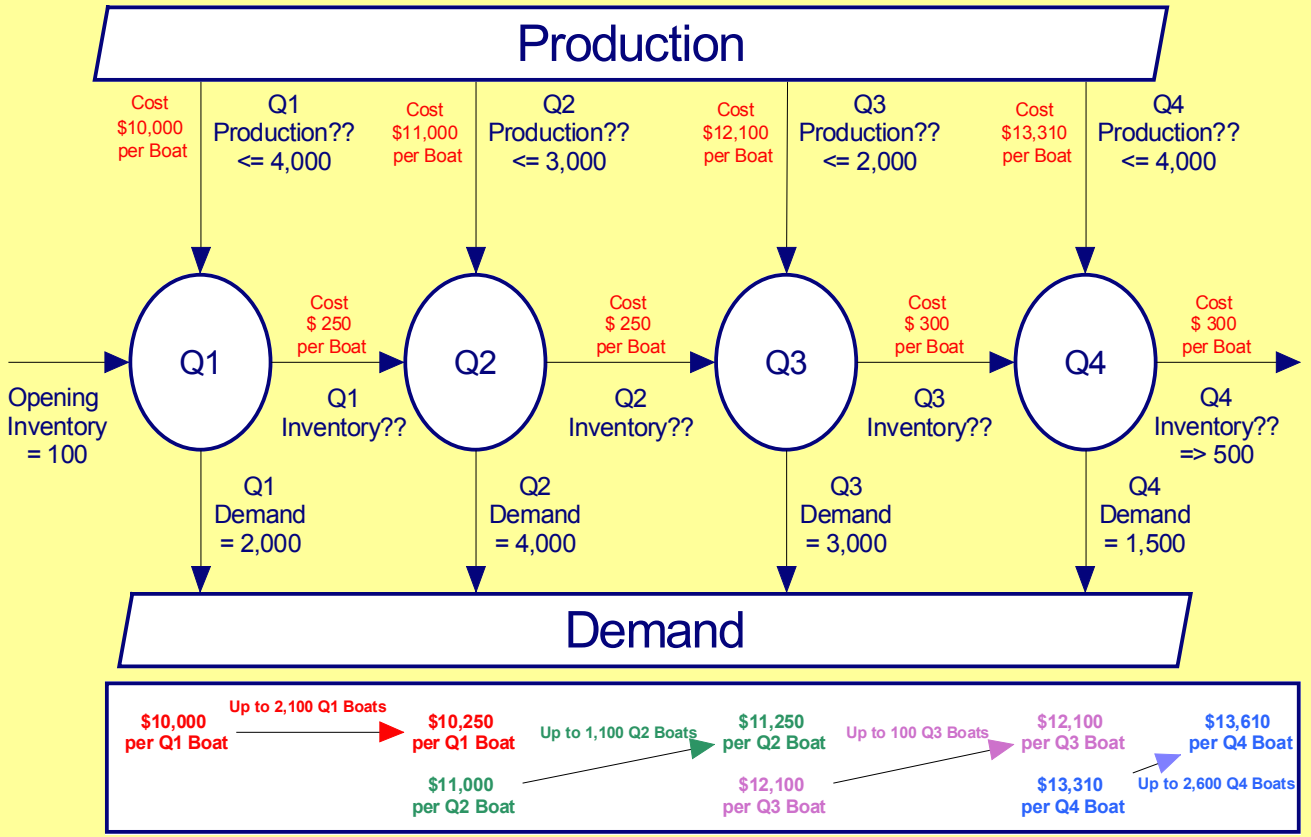
=B7*E7

=B14

=B7

=B13+B10-I10-B14

Another Look at The Diagram…



A Guess at a Solution…

- Q1Production = 4,000. Constraint Binding.
- Q2Production = 3,000. Constraint Binding.
- Q3Production = 2,000. Constraint Binding.
- Q4Production = 1,900. Constraint has slack of 2,100.
- Q1Inventory = 2,100.
- Q2Inventory = 1,100.
- Q3Inventory = 100.
- Q4Inventory = 500. Constraint Binding.

Excel's Answer Worksheet

Microsoft Excel 9.0 Answer Report
Worksheet: [DietProblem.xls]Inventory
Report Created: 1/26/2002 9:03:27 PM

Target Cell (Min)

Cell	Name	Original Value	Final Value
$B$3	CostMin	123,469,000	123,469,000

Adjustable Cells

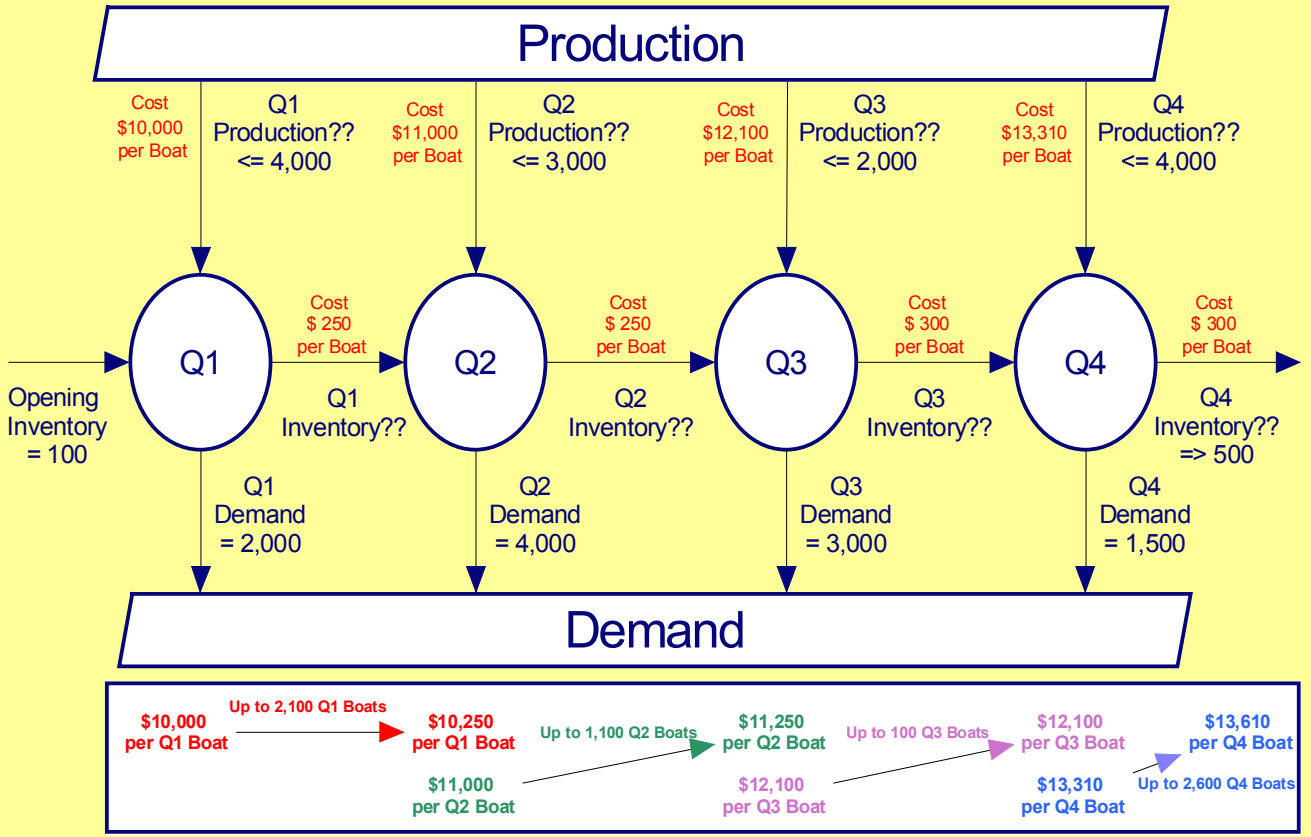
Cell	Name	Original Value	Final Value
$B$7	Q1Production	4000	4000
$B$8	Q2Production	3000	3000
$B$9	Q3Production	2000	2000
$B$10	Q4Production	1900	1900
$B$11	Q1Inventory	2100	2100
$B$12	Q2Inventory	1100	1100
$B$13	Q3Inventory	100	100
$B$14	Q4Inventory	500	500

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$B$18	InventoryQ4	500	$B$18>=$E$18	Binding	0
$B$19	ProductionQ1	4000	$B$19<=$E$19	Binding	0
$B$20	ProductionQ2	3000	$B$20<=$E$20	Binding	0
$B$21	ProductionQ3	2000	$B$21<=$E$21	Binding	0
$B$22	ProductionQ4	1900	$B$22<=$E$22	Not Binding	2100
$B$23	ConservationQ1	0	$B$23=$E$23	Binding	0
$B$26	ConservationQ4	0	$B$26=$E$26	Not Binding	0
$B$25	ConservationQ3	2.27374E-13	$B$25=$E$25	Not Binding	0
$B$24	ConservationQ2	0	$B$24=$E$24	Not Binding	0

- Our Prediction is borne out by Excel!!

A Further Look at The Diagram…



Excel's Sensitivity Report

Microsoft Excel 9.0 Sensitivity Report
Worksheet: [DietProblem.xls]Inventory
Report Created: 1/26/2002 9:03:27 PM

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$7	Q1Production	4000	0	10000	2510	1E+30
$B$8	Q2Production	3000	0	11000	1760	1E+30
$B$9	Q3Production	2000	0	12100	910	1E+30
$B$10	Q4Production	1900	0	13310	1E+30	910
$B$11	Q1Inventory	2100	0	250	2510	1E+30
$B$12	Q2Inventory	1100	0	250	1760	1E+30
$B$13	Q3Inventory	100	0	300	910	1E+30
$B$14	Q4Inventory	500	0	300	1E+30	13610

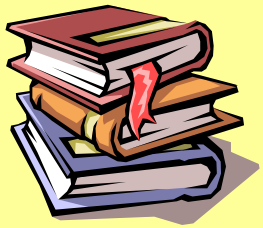
Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$B$18	InventoryQ4	500	13610	500	2100	500
$B$19	ProductionQ1	4000	-2510	4000	1900	100
$B$20	ProductionQ2	3000	-1760	3000	1900	100
$B$21	ProductionQ3	2000	-910	2000	1900	100
$B$22	ProductionQ4	1900	0	4000	1E+30	2100
$B$23	ConservationQ1	0	12510	0	100	1900
$B$26	ConservationQ4	0	13310	0	2100	1900
$B$25	ConservationQ3	2.27374E-13	13010	0	100	1900
$B$24	ConservationQ2	0	12760	0	100	1900

- Our Predictions are again correct!!!
- Note that Shadow Prices and Allowable Increases and Decreases are meaningless when it comes to the Conservation Constraints.

Reading and Homework.

- Read LP Example #6 – Transportation Handout.



- Homework #5 due in class on Thursday March 14th