

Linear Programs II

Risk Minimization and Marketing Mix Problems

January 8, 2002



FOMGT 353 Introduction to
Management Science



Risk Minimization Part a.

- Marks & Spenser. (M&S), is a brokerage firm that specializes in investment portfolios designed to minimize the risk exposure of its clients.
- A client who contacted M&S this past week has a maximum of \$50,000 to invest. M&S's investment advisor has decided to recommend a portfolio consisting of three investment funds: an International fund, a Blue Chip fund and a Government Bonds fund. The International fund has a projected annual return of 11%, the Blue Chip fund has a projected annual return of 9% and the Government Bonds fund has a projected annual return of 4%.
- The client wants to make at least \$4,000 per year but wishes to minimize risk overall.



Risk Minimization Part a. cont...

- M&S services include a risk rating for each investment alternative. The International fund, which is the most risky of the investment alternatives, has a risk rating of 9 per thousand dollars invested, the Blue Chip fund has a risk rating of 6 per thousand dollars invested and the Government Bonds fund a risk rating of 2. For example, if \$10,000 is invested in each of the investment funds, M&S's risk rating for the portfolio would be $9(10) + 6(10) + 2(10) = 170$.

a. What is the recommended investment portfolio for this client?

What is the risk rating for the portfolio?

Use the sensitivity report to predict the increase in the risk rating of the portfolio if the required return were increased to \$4,500.



Decision Variables

1.

2.

3.



Objective Function

- Is it min or max?
- What are the Coefficient Values?



Constraints

1.

2.



Bounds

- Borrowing, lending, shorting etc are not allowed, so the Decision Variables must be non-negative.



The Model

- Min 9 Int + 6 BlueChip + 2 GovtBond

s.t.

1. Int + BlueChip + GovtBond \leq 50,000
2. .11 Int + .09 BlueChip + .04 GovtBond \Rightarrow 4,000

Int, BlueChip, GovtBond \Rightarrow 0



The Model in Excel

Microsoft Excel - RiskMin.xls

File Edit View Insert Format Tools Data Window Help

G21 =

	A	B	C	D	E	F
1	Objective					
2						
3	Objective	0	$=((B7*E7)+(B8*E8)+(B9*E9))/1000$			
4						
5	Decision Variables			Return	Risk Factor	
6						
7	Int	0		0.11	9	
8	BlueChip	0		0.09	6	
9	GovtBonds	0		0.04	2	
10						
11						
12						
13						
14				$=(B7*D7)+(B8*D8)+(B9*D9)$		
15						
16	Constraints				Limit	
17						
18	Return	0			4000	
19	Investment	0			50000	
20				$=SUM(B7:B9)$		
21						



Results

Microsoft Excel 9.0 Answer Report
Worksheet: [RiskMin.xls]Parta
Report Created: 1/25/2002 3:30:53 PM

Target Cell (Min)

Cell	Name	Original Value	Final Value
\$B\$3	Objective	260	260

The Risk Rating

Adjustable Cells

Cell	Name	Original Value	Final Value
\$B\$7	Int	0	0
\$B\$8	BlueChip	40000	40000
\$B\$9	GovtBonds	10000	10000

The Portfolio \$40,000 in BlueChip
and \$10,000 in GovtBonds

Constraints

Cell	Name	Cell Value	Formula
\$B\$18	Return	4000	\$B\$18>=\$E\$18
\$B\$19	Investment	50000	\$B\$19<=\$E\$19

Microsoft Excel 9.0 Sensitivity Report
Worksheet: [RiskMin.xls]Parta
Report Created: 1/25/2002 3:28:42 PM

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
\$B\$7	Int	0	0.0014	0.009	1E+30	0.0014
\$B\$8	BlueChip	40000	0	0.006	0.001	0.0015
\$B\$9	GovtBonds	10000	0	0.002	0.000666667	0.0035

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
\$B\$18	Return	4000	0.08	4000	500	2000
\$B\$19	Investment	50000	-0.0012	50000	50000	5555.555556

The Allowable Increase for "Return" is 500, so we can increase to \$4,500 without changing the Decision Variables in the result. For every unit increase, the Shadow Price tells us that the Objective (Risk Rating will go up by 0.08 so an increase of \$500 will result in an increase of 40 to 300.



Risk Minimization Part b.

- b. Suppose that a second client with \$70,000 to invest wants to make at least \$7,000 per year. What is the recommended investment portfolio for this more aggressive investor? Discuss what happens to the portfolio as the required return increases. What can the Client do if they want a return of \$9,000?
- Has the problem changed?

OR are some of the parameters different?



The Model in Excel

Microsoft Excel - RiskMin.xls

File Edit View Insert Format Tools Data Window Help

B10 =

	A	B	C	D	E	F
1	Objective					
2						
3	Objective	8	=((B7*E7)+(B8*E8)+(B9*E9))/1000			
4						
5	Decision Variables			Return	Risk Factor	
6						
7	Int	0		0.11	9	
8	BlueChip	1000		0.09	6	
9	GovtBonds	1000		0.04	2	
10						
11						
12						
13						
14			=(B7*D7)+(B8*D8)+(B9*D9)			
15						
16	Constraints				Limit	
17						
18	Return	130			7000	
19	Investment	2000			70000	
20			=SUM(B7:B9)			
21						

- The model is unchanged!!
- The RHS of the constraints (Parameters!) have changed!!!



Results

Microsoft Excel 9.0 Answer Report
Worksheet: [RiskMin.xls]Partb
Report Created: 1/25/2002 3:55:56 PM

Target Cell (Min)

Cell	Name	Original Value	Final Value
\$B\$3	Objective	8	525

Adjustable Cells

Cell	Name	Original Value	Final Value
\$B\$7	Int	0	35000
\$B\$8	BlueChip	1000	35000
\$B\$9	GovtBonds	1000	0

Constraints

Cell	Name	Cell Value	Formula	Sta
\$B\$18	Return	7000	\$B\$18>=\$E\$18	Bind
\$B\$19	Investment	70000	\$B\$19<=\$E\$19	Bind

The Risk Rating

The Portfolio \$35,000 in Int and \$35,000 in BlueChip.

Microsoft Excel 9.0 Sensitivity Report
Worksheet: [RiskMin.xls]Partb
Report Created: 1/25/2002 3:55:56 PM

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
\$B\$7	Int	35000	0	0.009	1E+30	0.0014
\$B\$8	BlueChip	35000	0	0.006	0.001	1E+30
\$B\$9	GovtBonds	0	0.0035	0.002	1E+30	0.0035

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
\$B\$18	Return	7000	0.15	7000	700.0000001	700
\$B\$19	Investment	70000	-0.0075	70000	7777.777778	6363.636364

The Allowable Increase for "Return" is 700, so we can increase to \$7,700 (or 11%) without changing the Decision Variables in the result. As the required return increases, the amount in Int the riskier and more profitable investment will increase until the \$70,000 is all in Int at 11% for a "Return" of \$7,700.



Risk Minimization Part c.

c. Suppose that a third client with \$50,000 to invest wants the maximum return consistent with a maximum portfolio risk rating of 160. Develop the recommended investment portfolio for this investor. Discuss the interpretation of the slack variable for the total investment fund constraint. ?

- Has the problem changed?

OR are some of the parameters different?



Risk Minimization

Part c. cont...

- Yes, the problem has changed.
 - The client wants to maximize Return,

- subject to a maximum portfolio risk rating of 160.

	A	B	C	D	E	F
1						
2	Objective	0	$=(B6*D6)+(B7*D7)+(B8*D8)$			
3						
4	Decision Variables			Return	Risk Factor	
5						
6	International	0		0.11	9	
7	BlueChip	0		0.09	6	
8	GovtBonds	0		0.04	2	
9						
10	Constraints				Limit	
11						
12	Risk	0			160	
13	Investment	0			50000	
14						
15						
16						



Results

Microsoft Excel 9.0 Answer Report
Worksheet: [RiskMin.xls]Partc
Report Created: 1/25/2002 4:22:48 PM

Target Cell (Max)

Cell	Name	Original Value	Final Value
\$B\$2	Objective	0	2750

The Return.

Adjustable Cells

Cell	Name	Original Value	Final Value
\$B\$6	International	0	0
\$B\$7	BlueChip	0	15000
\$B\$8	GovtBonds	0	35000

The Portfolio \$15,000 in BlueChip and \$35,000 in GovtBonds.

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
\$B\$12	Risk	160	\$B\$12<= \$E\$12	Binding	0
\$B\$13	Investment	50000	\$B\$13<= \$E\$13	Binding	0

9.0 Sensitivity Report
[RiskMin.xls]Partc
1/25/2002 4:22:48 PM

Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
\$B\$6 International	0	-0.0175	0.11	0.0175	1E+30
\$B\$7 BlueChip	15000	0	0.09	0.03	0.01
\$B\$8 GovtBonds	35000	0	0.04	0.023333333	0.01

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
\$B\$12	Risk	160	12.5	160	140	60
\$B\$13	Investment	50000	0.015	50000	30000	23333.33333

The zero value of the slack for the Investment Constraint implies that the total amount invested equals the right hand side value of the constraint which accordingly is binding. The shadow price for Investment tells us that for each extra dollar available for investment we would make an extra 1.5 cents with an allowable increase of \$30,000.



Marketing Mix Problem

- The Restaurant at the End of the Universe would like to determine the best way to allocate a monthly advertising budget of \$1,000 between newspaper advertising, radio advertising and billboards.
- Management has decided that at least 15% of the budget must be spent on each type of media, and that the amount of money spent on local newspaper advertising must be at least twice the amount spent on radio advertising.
- A marketing consultant has developed an index that measures audience exposure per dollar of advertising on a scale from 0 to 100, with higher values implying greater audience exposure. The value of the index for local newspaper advertising is 50, the value of the index for spot radio advertising is 80 and for billboards the index is 75
- How should the restaurant allocate its advertising budget in order to maximize the value of total audience exposure?
 - a. Formulate an LP model to determine how the restaurant should allocate its advertising budget.
 - b. Solve the problem using Excel or some other computer program.



The Decision Variables and the Objective Function

- Decision Variables:

- The Objective Function:



Constraints

- 1.
- 2.
- 3.
- 4.
- 5.



The Model

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Excel

	A	B	C	D	E	F	G	H
1								
2		Objective						
3								
4		MaxExposure	38,250					
5								
6		Decision Variables			Advertisizing Index			
7								
8		News	300		50			
9		Radio	150		80			
10		Bilboard	150		75			
11								
12		Constraints						
13								
14		Budget	600		1000			
15		MinNewspaper	300		150			
16		MinRadio	150		150			
17		MinBillboard	150		150			
18		Newspaper Radio	0		0			
19								
20								
21								

Solver Parameters

Set Target Cell:

Equal To: Max Min Value of:

By Changing Cells:

Subject to the Constraints:

-
-
-
-
-



Results

Microsoft Excel 9.0 Answer Report
 Worksheet: [Marketing.xls]MarketingMix
 Report Created: 1/25/2002 5:19:55 PM

Target Cell (Max)

Cell	Name	Original Value	Final Value
\$C\$4	MaxExposure	38,250	68,250

Adjustable Cells

Cell	Name	Original Value	Final Value
\$C\$8	News	300	300
\$C\$9	Radio	150	150
\$C\$10	Bilboard	150	550

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
\$C\$14	Budget	1000	\$C\$14<=\$E\$14	Binding	0
\$C\$15	MinNewspaper	300	\$C\$15>=\$E\$15	Not Binding	150
\$C\$16	MinRadio	150	\$C\$16>=\$E\$16	Binding	0
\$C\$18	Newspaper Radio	0	\$C\$18>=\$E\$18	Binding	0
\$C\$17	MinBillboard	550	\$C\$17>=\$E\$17	Not Binding	400

- \$300 on Newspaper ads.
- \$150 on Radio ads.
- \$550 on Billboards
- Gives 68,250 audience Exposure value.



Reading and Homework.

- Read Chapter 3, Section 3.4
- Read LP Example #4 - The Diet Problem Handout.
- Read LP Example #5 - Inventory Holding Over Time Periods Handout.

