UNIVERSITY OF MASSACHUSETTS

Isenberg School of Management

Department of Finance and Operations Management

FOMGT 353-Introduction to Management Science

Practice Exam #1 Answers

Show your work completely and in an organized manner to receive maximum credit. Correct answers without supporting calculations or diagrams will not receive credit. Incorrect answers using the correct method and a good presentation will receive substantial credit.

My name is:

There are 5 questions each worth 20 points each pay attention to the point scoring as it may help you in getting to a good answer.

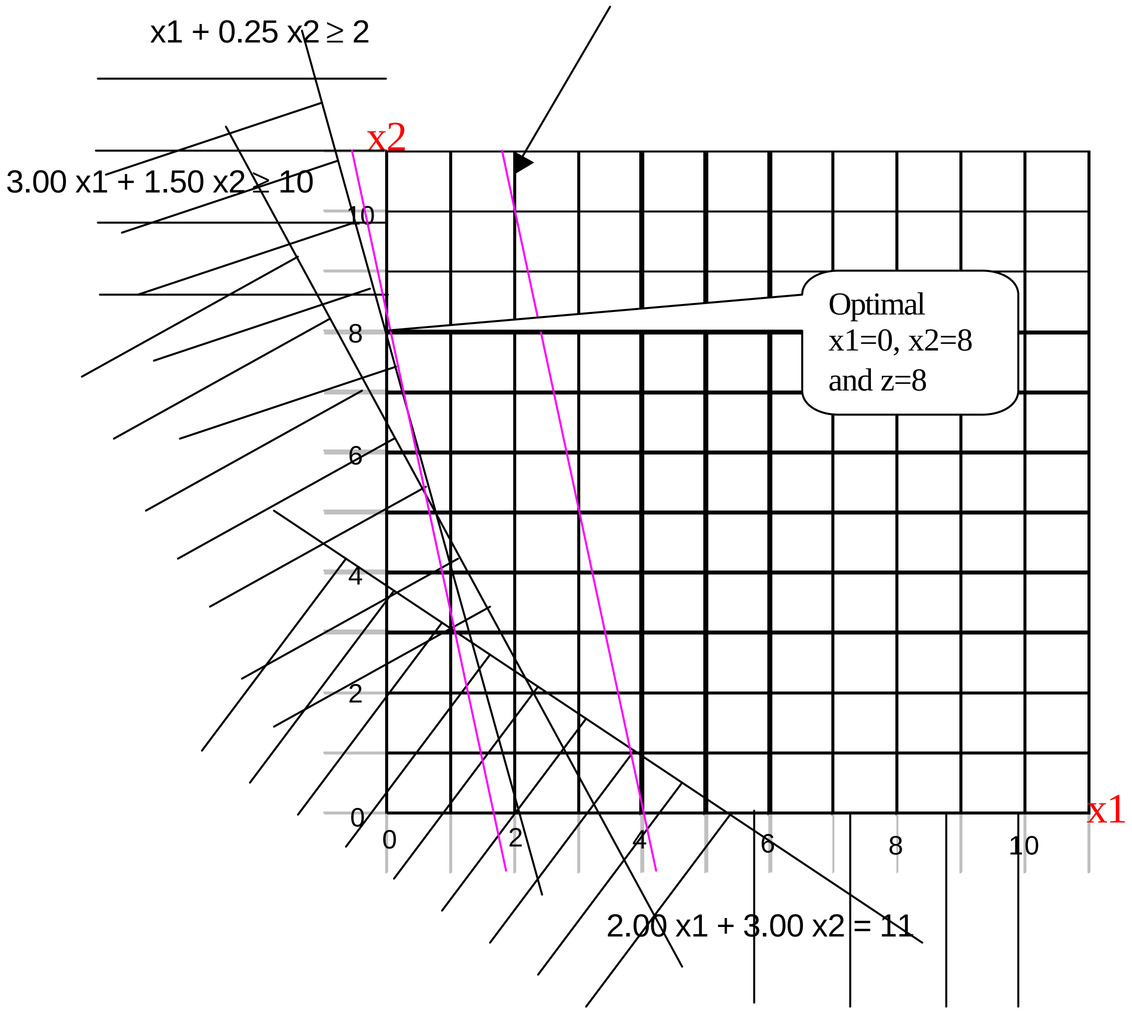
Question 1. – Use the Graphical Method to solve the following linear program.

(2 points per correctly drawn constraint or bound, 3 points for correctly identifying the feasible region, 3 points for using the objective function to draw a line or series of lines, which correctly identify the optimal point or points and 4 points for stating the optimal solution)

Min 5 x1 + x2
s.t.
2.00 x1 + 3.00 x2 ≥ 11 (1)
3.00 x1 + 1.50 x2 ≥ 10 (2)
x1 + 0.25 x2 ≥ 2 (3)
x1, x2 ≥ 0

(1) x1=0, then x2=11/3=3.66 :x2=0, then x1= 11/2 =5.5
(2) x1=0, then x2=10/1.5=6.66 :x2=0, then x1= 10/3=3.33
(3) x1=0, then x2= 2/0.25=8 :x2=0, then x1= 2/1=2

Use 5x1 + x2 = 10 (say) to graph the Objective Function gradient.
x1=0, then x2= 10 :x2=0, then x1= 10/5=2



Question 2 - Formulation of a Linear Program.

(1 point for correctly identifying each Decision variable and 1 point for declaring each variable used in the problem, succinctly. 2 Points for correctly stating the Objective Function 3 points for each correct constraint and 3 point for the proper form and bounds to the problem)

Horatio's cookies are famous throughout Southern Ohio. Each 2 lb box of cookies provides a dollar contribution to profit. Horatio has 1,000 lbs of sugar, 1,500 lbs of flour and 800 lbs of chocolate. Horatio is trying to decide how to get the most contribution from making Choc Chip Cookies, Walnut Cookies and Vanilla Cookies.

A box of Choc Chip takes up 1lb of flour, 8 ounces of sugar and 8 ounces of chocolate. A box of Walnut cookies uses 1.2 lbs of flour and a box of Vanilla cookies uses 1.5 lbs of flour. Strangely Walnut cookies contain no walnuts, but do use 0.75 lbs of sugar and 0.25 lbs of chocolate for each 2lb box. Equally odd is the absence of vanilla from Vanilla Cookies, which take 0.4 lbs of chocolate and 0.6 lbs of sugar per 2 lb box.

Horatio cannot borrow or locate any extra ingredients and he cannot make negative quantities of 2 lb boxes of cookies.

Formulate a linear program to address Horatio's production planning dilemma.

Let $x1$ be the number of 2lb boxes of Choc Chip Cookies produced
Let $x2$ be the number of 2lb boxes of Walnut Cookies produced
Let $x3$ be the number of 2lb boxes of Vanilla Cookies produced

Max $x1 + x2 + x3$
s.t.
$1\ x1 + 1.2\ x2 + 1.5\ x3 <= 1{,}500$ (Flour)
$\frac{1}{2}\ x1 + \frac{3}{4}\ x2 + 0.6\ x3 <= 1{,}000$ (Sugar)
$\frac{1}{2}\ x1 + \frac{1}{4}\ x2 + 0.4\ x3 <= 800$ (Chocolate)
$x1, x2, x3 => 0$

Question 3 – A Simplex Pivot

(2 points for identifying the entering variable and 2 points for correct statement of the rule used. 2 points for the statement of the Ratio Test and 2 points for correctly identifying the leaving variable. 6 points for pivoting correctly, 2 points for updating the Zj values and the Z value, and 2 points for updating the Cj-Zj values. 2 points for stating the optimal solution in full.)

If the original Linear Program were:

Max 0.30 x1 + 0.15 x2
Subject to
x1 + 0.35 x2 <= 100 (Molasses)
0.3 x1 + 0.20 x2 <= 36 (Cocoa)
0.2 x1 + 0.50 x2 <= 60 (Milk)
x1, x2 => 0

And the Tableau before the last pivot is:

		x1	x2	s1	s2	s3	
Basis	Cj	0.3	0.15	0	0	0	Bi
x1	0.3	1	0.35	1	0	0	100
s2	0	0	0.095	-0.3	1	0	6
s3	0	0	0.43	-0.2	0	1	40
	Zj	0.3	0.105	0.3	0	0	30
	Cj - Zj	0	0.045	-0.3	0	0	

Pivot, identifying all of the appropriate steps and rules used, to the optimal solution and state what the optimal solution is. Note the points allocation for this problem. Expect simpler math than division by 0.095 in the real exam!!

In a maximization problem the entering variable is the variable whose (Cj-Zj) value is the most positive number, so in this case x2 enters.

The Ratio Test values are the Bi divided by the coefficients in the Pivot Column or 100/0.35 = 285.7; 6/0.095=63.1 and 40/0.43=93.0. The Ratio Test requires the leaving variable to be the one whose Ratio Test Value is a positive number and the smallest of the positive Ratio Test Values. In this case 63.1 is the smallest positive Ratio Test Value and so s2 is the Leaving Variable.

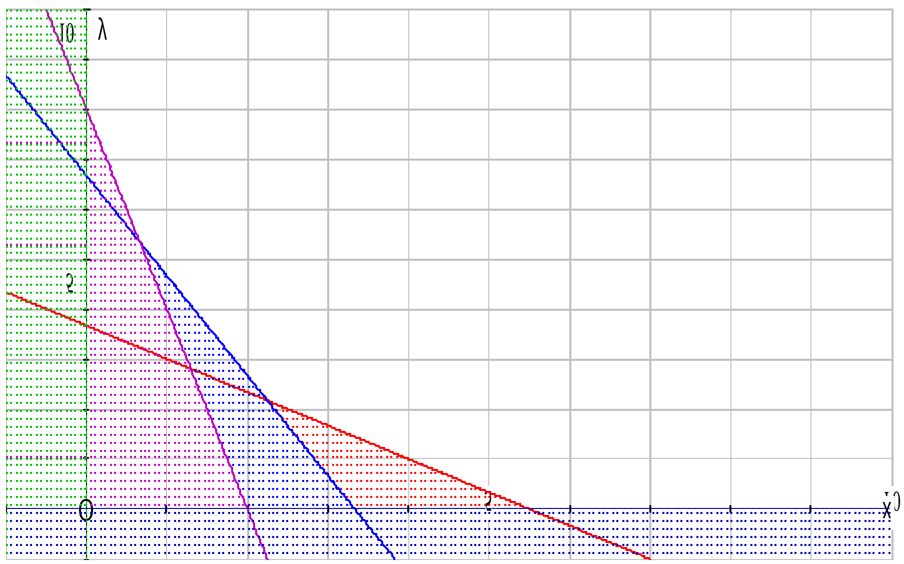
We did this pivot in HW3 so refer to the solution to that homework for the answer to the rest of this Question.

Basis	Cj						Bi
	Zj						
	Cj - Zj						

Question 4. - Special cases.

(There a 4 sub-parts to this question, 4 points in each for identifying and describing the special case and for stating the remedy or approach appropriate.)

i. You have drawn the following graph and you are told that the objective function is Maximize x1 + x2



This problem is unbounded. (x1 + x2 can be increased infinitely without hitting a constraint).

ii. You have reached the following tableau while using the Simplex Algorithm to solve a maximization problem.

		x1	x2	s1	s2	a2	s3			
	Cj	2	1	0	0	-1000	0	bi		
s1	0	0	0	1	-0.25	0.25	0.75	4.75		-19
x2	1	0	1	0	-0.5	0.5	-0.5	1.5		-3
x1	2	1	0	0	-0.5	0.5	0.5	3.5		-7
	Zj	2	1	0	-1.5	1.5	0.5			
	Cj-Zj	0	0	0	1.5	-1002	-0.5		1.5	

Where all the ratio test values are negative the problem is unbounded.

iii. You have reached the following tableau while using the Simplex Algorithm to solve a minimization problem.

		x1	x2	s1	s2	a2	s3			
	Cj	1	1	0	0	1000	0	bi		
x2	1	0	1	0.667	-0.67	0.667	0	4.666667		7
x1	1	1	0	-0.67	-0.33	0.333	0	0.333333		-0.5
s3	0	0	0	1.333	-0.33	0.333	1	6.333333		4.75
	Zj	1	1	0	-1	1	0	5.000		
	Cj-Zj	0	0	0	1	999	0			

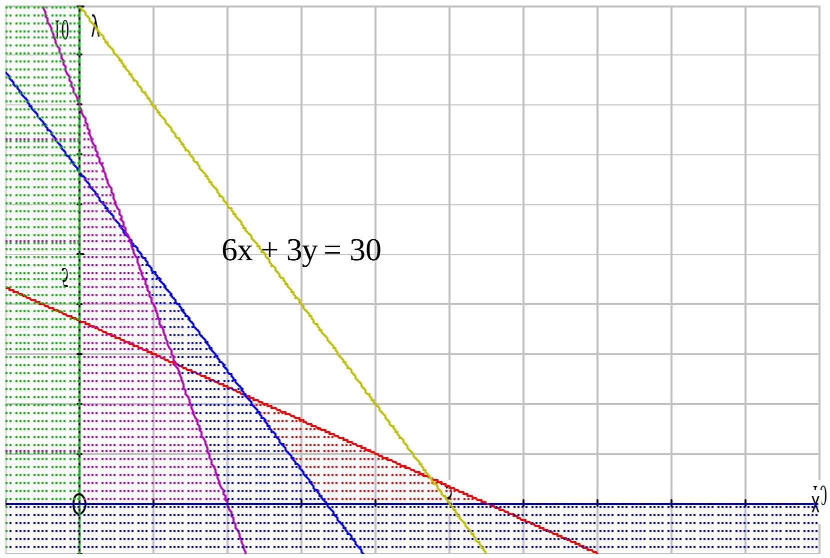
In a minimization, I would have an entering variable if I had a negative (Cj-Zj) value. There is no negative (Cj-Zj) value in this Tableau, so the solution is an optimal solution. Since there are 3 variables in the basis, but 4 (ie more than the number of variables in the basis) 0 valued (Cj-Zj)'s, there are multiple optimal solutions to this problem.

iv. If this is your final tableau, what can you say about the nature of the problem?

		x1	x2	s1	s2	a2	
		1	1	0	0	-1000	
x1	1	1	0	1	0	0	2
x2	1	0	1	0	1	1	0
Zj		1	1	1	1	1	2
(Cj-Zj)		0	0	-1	-1	-1001	

Since one of the basis variables is set to 0, we know that this problem is degenerate.

v. If your constraints look like the following and the Objective Function is 6x + 3y, what is the nature of the solution?



The slope of the Objective Function is the same as the slope of the "Blue" constraint and so there are multiple optimal solutions to this problem.

Question 5. The model developed in Question 2 was solved using Excel and the Answer and Sensitivity Sheets are as follows:
(1 point for each decision variable value correctly stated, 2 points for each slack variable value correctly stated and 1 point for the Objective Function Value)

Target Cell (Max)

Cell	Name	Original Value	Final Value
$C$4	MaxContribution	0.00	2000.00

Adjustable Cells

Cell	Name	Original Value	Final Value
$C$8	ChocChip	0	-2.27374E-13
$C$9	Walnut	0	0
$C$10	Vanilla	0	2000

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$C$14	MaxSugar	1000	$C$14<=$E$14	Binding	0
$C$15	MaxFlour	500	$C$15<=$E$15	Not Binding	1000
$C$16	MaxChocolate	800	$C$16<=$E$16	Binding	0

State the optimal solution, giving the values of the Decision variables, the slack variables and the Objective Function Value.

Horatio should make 0 boxes of Choc Chip Cookies (or –2.27374E-13), 0 boxes of Walnut Cookies, 2,000 boxes of Vanilla Cookies. There is no sugar left over and no chocolate left over, but there is 1,000 pounds of Flour unused. The contribution to profits will be $2,000.

The sensitivity report is as follows:

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$C$8	ChocChip	-2.27374E-13	0	1	0.5	0
$C$9	Walnut	0	-1	1	1	1E+30
$C$10	Vanilla	2000	0	1	0	0.333333333

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$C$14	MaxSugar	1000	2	1000	0	333.3333333
$C$15	MaxFlour	500	0	1500	1E+30	1000
$C$16	MaxChocolate	800	0	800	400	0

By how much could the contribution expected from each 2 lb box of vanilla cookies be reduced before there would be a change to the basis? (2.5 points)

A reduction of 33.333 cents in the contribution expected from each 2lb of vanilla cookies would result in a change to the basis.

By how much would the contribution from each 2 lb box of Choc Chip cookies have to increase before it was worth making Choc Chip cookies. (2.5 points)

It would be worth making some Choc Chip Cookies if the Contribution from these cookies were more than $1.50

If Horatio could get some more flour would this improve his contribution to profits? (2.5 points)

Horatio already has more flour than he knows what to do with (1,000 lbs left over [slack]). Having more flour would not enable Horatio to make a higher contribution to profit and this is confirmed by the 0 shadow price for flour.

What does 1E+30 mean? (2.5 points) 1 E+30 is a very large number and substitutes for infinity in Excel.

Bonus Question

(10 points)

In the sensitivity report sugar has a shadow price of $2, but an allowable increase of 0, what does this mean?   How does this relate to the “Final Value” for Choc Chip cookies of –2.27374E-13?

Excel is failing to provide a good number for the number of Choc Chip Cookie boxes to be made. Between minus 2.27374E-13 and zero, we would see an increase in the contribution to profit, from the production of an extra box of cookies. Since from minus 2.2734E-13 to 0 is essentially zero, the allowable increase is 0!!