# Arrivals:

**Jobs** arrive for **service**.

***Parameter*** is **Mean Arrival Rate - λ** (lamda) (so many jobs arriving per period)

***Key Statistic*** is the **Interarrival Time**, the average time between the arrivals of a job.

Average time between arrival of jobs = mean interarrival time = **1/λ**

**This statistic can be:**

- **D**eterministic (1 every time period, without fail) (Use Simulation Methods)
- According to a Probability Distribution:
  - **M**arkovian
  - **G**eneral (Use Simulation Methods)

## Markov Processes (Chains)

"A process consisting of a countable sequence of stages, that can be judged at each stage to fall into future states independent of how the process arrived at the previous state."

"**Memoryless**" A property of a probability distribution that means that the probabilities of future events are unchanged by the passage of time.

# Service:

**Jobs** are **serviced**.

***Parameter*** is **Mean Service Rate - μ** (mu) (So many jobs serviceable per period)

***Key Statistic*** is the **Mean or Average Potential Service Time,** the average time it takes to service a job.

Average Potential Service Time = **1/μ**

**This statistic can be:**

- **D**eterministic (1 every time period, without fail)
- According to a Probability Distribution:
  - **M**arkovian
  - **G**eneral (we only use μ and σ)

**Number of servers** – can be **one** or **many**.

## Queues

- Queue Length – may be limitless (infinite) or finite.
- Job (customer) Population - may be infinite of finite.
- Queue Discipline – jobs are served on FIFO.
- Time Horizon – system foes on an on over and infinite or finite time frame.

## Warning if $\lambda \geq \mu$ none of this works

# Factors in Queue Design

- Line configuration one long one or several smaller ones.
- Line Switching (Jockeying)
- Balking (Lost Jobs / Customers
- Priority (Do I want to server all jobs on FIFO – airlines!!)
- Tandem Queues (one service, then another)
- Homogeneity of Service.

# Performance Measurement

$P_0$ - Probability that there are no jobs in the system.

$P_n$ - Probability that there are n jobs in the system.

L - Average number of jobs in the system.

$L_q$ - Average number of jobs in the queue.

W - Average time a job spends in the system.

$W_q$ - Average time a job spends in the queue.

$P_w$ - Probability that all servers are busy.

ρ - The Utilization rate for each server (percentage of time each server is busy).

# Queue Classification

**Arrival Process / Service Process / Number of Servers / Finiteness of Queue Length / Finiteness of Job Population.**

- **M =** exponential and reverse exponential (Poisson) distribution
- **D =** deterministic
- **G =** any general distribution of service times
- **1 =** 1 server
- **k =** more than one server
- **F=** total number of potential jobs

**Which gives us a number of choices to look at:**

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| M | / | M | / | 1 | / | | | |
| M | / | M | / | k | / | | | |
| M | / | G | / | 1 | | | | |
| M | / | D | / | 1 | | | | |
| M | / | M | / | k | / | F | | |
| M | / | M | / | 1 | / | F | λ | ≠μ |
| M | / | M | / | 1 | / | F | λ | =μ |
| M | / | M | / | 1 | / | | / | m |